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Statistical Energy Analysis as a Tool for Quantifying Sound and Vibration Transmission Paths

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Statistical energy analysis as a tool for quantifying sound and vibration transmission paths

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When a complex structure is excited in several different ways by different sources, the SEA energy balance equations result in a set of linear equations that can be used to calculate loss factors, coupling loss factors or net energy flows and incoming powers. If certain symmetry relations are used, and/or if some prior knowledge about the system is available, the set of linear equations is overdetermined and can be solved by a least square technique.

A good indicator for the direction of the energy flow is the SEA temperature of the subsystems.

Experiments and computer simulations performed on three plate arrangements gave in general good results when the coupling was weak and there were more than three modes in the frequency band of interest. Not so good results were obtained when a small energy flow has to be measured as the difference of large quantities.

1. Introduction

Statistical energy analysis (SEA) was developed more than thirty years ago by Lyon, Smith, Maidanik, and others as a tool for predicting the mean square velocities of thin space-craft or aircraft structures when they are excited by sources (jet noise, turbulent boundary layer, etc.) that are random in nature and therefore contain wide frequency bands. A comprehensive description of the basic ideas, and some applications of SEA, is given in a book by Lyon (1975).

As its forerunner, the heat conduction model for the vibration distribution in buildings (Westphal 1957), SEA consists of a system of linear equations that describe the energy flow between substructures of a complex system. Because those equations are manifestations of the law of conservation of energy they are very robust. Quite often they give good results even if the usual requirements for their validity (large number of modes, sufficient modal overlap, etc.) may be violated.

As SEA has been successful in the prediction of average vibration amplitudes and sound pressures in space vehicles, airplanes, ships, buildings, large machines, etc., it certainly is worthwhile to try to use it as a tool for solving the 'inverse problem'; i.e. to investigate the energy flow and the coupling properties in existing structures. This would be of considerable help for optimizing noise control. It would allow us to find those paths that are responsible for the sound transmission in complex arrangements. It also would be of considerable help in the appropriate design of additional damping and isolation.

2. Possible applications of the 'inverse' SEA

In buildings, ships, vehicles, machines, etc., the sound is very often caused by several sources and is transmitted along different paths. Because the sound powers

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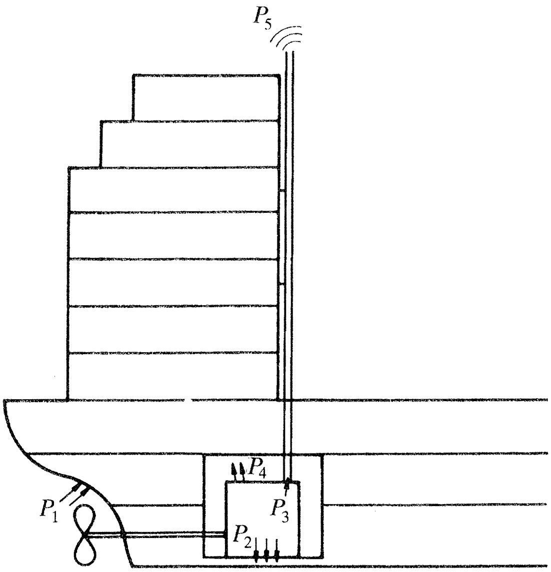

Figure 1. Major sources of acoustic power in a ship.

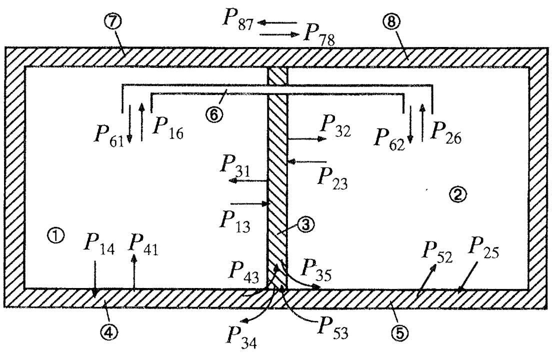

Figure 2. Acoustic energy flow between two adjacent rooms in a building. 1, Air-filled space (source room); 2, air-filled space (receiver room); 3, partition; 4, 5, 7, 8, flanking walls; 6, air duct. The arrows indicate the energy flow (not all possibilities are shown).

that are transmitted into a structure, appear explicitly in the SEA equations, it should be possible to determine them if all the other quantities in the equations are known. Thus if one wants, for example, to measure (see figure 1) the powers that are generated by a ship propeller (P_1), the main engine vibrations (P_2), the exhaust pipe vibrations (P_3), the air-borne sound from the engine (P_4), the radiated exhaust noise (P_5), one should be able to do so by determining the mechanical energies

$$E_\nu = m_\nu \overline{v_\nu^2} \quad \text{or} \quad E_\nu = (V_\nu/\rho c)\, \overline{p_\nu^2} \tag{1}$$

in certain parts of the ship and inserting them into the SEA equations. In (1) E_ν is the mechanical energy in the νth subsystem of mass m_ν or volume V_ν. The parameters ρ, c are the density and speed of sound in the air- or liquid filled volume V_ν. The value $\overline{v_\nu^2}$ is the mean square velocity of the νth subsystem and $\overline{p_\nu^2}$ the mean square sound pressure. Obviously the powers can only be measured if all the loss factors and coupling loss factors that appear in the SEA equations are known (which is hardly ever the case).

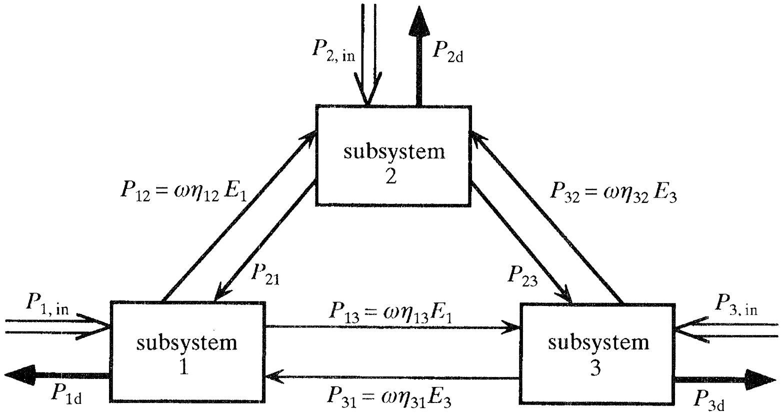

Figure 3. Idealization of a structure consisting of three subsystems (see equation (2)).

Figure 2 shows an example from building acoustics. The aim in this case may be to quantify the powers $P_{k\nu}$ that flow from one system to another to find out where sound is mainly transmitted and where any further isolation is most effective.

In this paper we are interested in the transmission paths; i.e. the energy flow quantities $P_{k\nu}$ which in standard SEA are usually expressed in terms of the coupling loss factors $\eta_{k\nu}$.

To measure these quantities in a three subsystem arrangement (see figure 3) we can make the following sequence of experiments.

In the first experiment subsystem 1 is excited by a stationary random power $P_1^{(1)}$ and the average energies $E_\nu^{(1)}$ are measured in each subsystem ($\nu = 1, 2, 3$). In the second experiment subsystem 2 is excited by $P_2^{(2)}$ and $E_\nu^{(2)}$ is determined. In a third experiment one proceeds in a similar way. This allows us to set up the following equation:

$$\left.\begin{aligned} &\eta_{\nu d} E_\nu^{(\mu)} + \sum_{k=1,\, \neq \nu}^{3} (\eta_{\nu k} E_\nu^{(\mu)} - \eta_{k\nu} E_k^{(\mu)}) = \frac{1}{\omega} P_{\nu,\,\mathrm{in}}^{(\mu)} \delta_{\mu\nu} \\ \text{for} \quad &\nu = 1,2,3, \quad \mu = 1,2,3, \quad \delta_{\nu\nu} = 1; \quad \delta_{\nu\mu} = 0 \quad \text{for} \quad \nu \neq \mu. \end{aligned}\right\} \tag{2}$$

Here the superscript indicates the number of the experiment and the subscript gives the number of the subsystem. ω is the angular frequency, $\eta_{1d}, \eta_{2d}, \eta_{3d}$ are the loss factors that characterize powers

$$P_{\nu d} = \omega \eta_{\nu d} E_\nu \tag{3}$$

lost in each system. $\eta_{\nu k}$ are the coupling loss factors; they determine the energy flow (power) from the νth subsystem to the k-subsystem; i.e.

$$P_{\nu k} = \omega \eta_{\nu k} E_\nu. \tag{4}$$

As (2) consists of nine linear equations, one might argue that it allows to determine nine unknown quantities; i.e. the six coupling loss factors $\eta_{\nu k}$, and the three internal loss factors $\eta_{\nu d}$ or the $\eta_{\nu k}$ values and the input powers $P_\mu^{(\mu)}$, etc.

If the number of subsystems is not three, the same procedure which always give n^2 linear equations can be applied (n = number of subsystems).

3. Short literature survey

The idea to use the SEA-equations to determine coupling loss factors is not new.

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Apart from some general remarks in Lyon's book, Bies & Hamid (1980) seem to have been the first to use SEA in an inverse way. They realized that the accuracy of the method is not very high, and therefore used an overdetermined system which they solved by an error minimization technique.

Woodhouse (1981) also discussed the accuracy problem. He suggested modifying the measured data in such a way that the final results do not violate the basic conservation law (i.e. no negative coupling loss factors are allowed). Obviously the modifications of the measured data were kept to a minimum. The numerical procedures for this method are described in more detail by Hodges *et al.* (1987). Clarkson & Ranky (1984) also found that *in situ* measurements of internal and coupling loss factors are of limited accuracy. They suggested an iteration scheme to improve the results.

Lalor *et al.* (1989, 1990) combined the equations in (2) to get

$$\left.\begin{aligned} \eta_{21}\left(\frac{E_2^{(2)}}{E_1^{(2)}}-\frac{E_2^{(1)}}{E_1^{(1)}}\right)+\eta_{31}\left(\frac{E_3^{(2)}}{E_1^{(2)}}-\frac{E_3^{(1)}}{E_1^{(1)}}\right)&=\frac{P_1^{(1)}}{\omega E_1^{(1)}},\\ \eta_{21}\left(\frac{E_2^{(3)}}{E_1^{(3)}}-\frac{E_2^{(1)}}{E_1^{(1)}}\right)+\eta_{31}\left(\frac{E_3^{(3)}}{E_1^{(3)}}-\frac{E_3^{(1)}}{E_1^{(1)}}\right)&=\frac{P_1^{(1)}}{\omega E_1^{(1)}}.\end{aligned}\right\} \tag{5a}$$

In a similar way they found

$$\left.\begin{aligned} \eta_{12}\left(\frac{E_1^{(1)}}{E_2^{(1)}}-\frac{E_1^{(2)}}{E_2^{(2)}}\right)+\eta_{32}\left(\frac{E_3^{(1)}}{E_2^{(1)}}-\frac{E_3^{(2)}}{E_2^{(2)}}\right)&=\frac{P_2^{(2)}}{\omega E_2^{(2)}},\\ \eta_{12}\left(\frac{E_1^{(3)}}{E_2^{(3)}}-\frac{E_1^{(2)}}{E_2^{(2)}}\right)+\eta_{32}\left(\frac{E_3^{(3)}}{E_2^{(3)}}-\frac{E_3^{(2)}}{E_2^{(2)}}\right)&=\frac{P_2^{(2)}}{\omega E_2^{(2)}}\end{aligned}\right\} \tag{5b}$$

and

$$\left.\begin{aligned} \eta_{13}\left(\frac{E_1^{(1)}}{E_3^{(1)}}-\frac{E_1^{(3)}}{E_3^{(3)}}\right)+\eta_{23}\left(\frac{E_2^{(1)}}{E_3^{(1)}}-\frac{E_2^{(3)}}{E_3^{(3)}}\right)&=\frac{P_3^{(3)}}{\omega E_3^{(3)}},\\ \eta_{13}\left(\frac{E_1^{(2)}}{E_3^{(2)}}-\frac{E_1^{(3)}}{E_3^{(3)}}\right)+\eta_{23}\left(\frac{E_2^{(2)}}{E_3^{(2)}}-\frac{E_2^{(3)}}{E_3^{(3)}}\right)&=\frac{P_3^{(3)}}{\omega E_3^{(3)}}.\end{aligned}\right\} \tag{5c}$$

By adding the equations for $\mu = 1,2,3$ in (2) one finds for the internal loss factors

$$\left.\begin{aligned} \eta_{1d}E_1^{(1)}+\eta_{2d}E_2^{(1)}+\eta_{3d}E_3^{(1)}&=P_1^{(1)}/\omega,\\ \eta_{1d}E_1^{(2)}+\eta_{2d}E_2^{(2)}+\eta_{3d}E_3^{(2)}&=P_2^{(2)}/\omega,\\ \eta_{1d}E_1^{(3)}+\eta_{2d}E_2^{(3)}+\eta_{3d}E_3^{(3)}&=P_3^{(3)}/\omega.\end{aligned}\right\} \tag{5d}$$

This way the coupling loss factors are separated from the internal loss factors and the equations that have to be solved are much simpler. Ming *et al.* (1990) applied this method to a car and found very reliable results.

Obviously the procedure underlying (5*a*–*d*) can also be applied to subsystems that are excited in n different ways. In this case the basic $n\times n$ equations can be rearranged so that there are n sets of $(n-1)$ equations containing the $n-1$ coupling loss factors. In addition there are n equations containing only the internal loss factors. Measuring all the n^2+n coefficients that are needed in these equations is a very formidable task, because for each frequency band averages have to be taken over the surface of each subsystem and also over several excitation points on each subsystem.

The rearrangement of (2) proposed by Lalor *et al.* simplifies the equations somewhat, but it also shows that for a strongly coupled system the method becomes very sensitive to the slightest error. When a system consists of strongly coupled subsystems it vibrates more or less the same way whichever subsystems are excited. Thus the ratio of energies in different experiments is practically equal – e.g. $E_1^{(1)}/E_2^{(1)} \approx E_1^{(2)}/E_2^{(2)}$ – and therefore the elements of the matrices of (5*a*–*c*) become almost zero.

4. The thermodynamic analogy as an indicator for the direction of energy flow

In the early publications on SEA it was mentioned already that there exists a thermodynamic analogy where the energy per mode corresponds to temperature and the coupling coefficient (which is not identical with the coupling loss factor) corresponds to the heat conduction coefficient. With this analogy in mind one can easily find the direction of energy flow in a construction composed of many multimodal substructures, because energy always flows from the higher temperature to the lower one. It is also obvious that subsystems are strongly coupled if their temperatures are equal or almost equal.

To apply this general idea it is necessary to find the SEA 'temperature' of the νth substructure. It is given by

$$T_\nu = E_\nu/\Delta N_\nu. \tag{6}$$

Here the energy E_ν is given by (1) and ΔN_ν is the number of modes of the ν-subsystem within the frequency band of interest.

The measurement of the energies E_ν can be done by standard techniques; the number of modes has to be estimated somehow. One method to obtain ΔN_ν would be to measure the number of resonances for each subsystem in the frequency bands of interest. When there are not too many resonances (average distance between two resonances larger than three times the bandwidth of a resonance) this may give a reasonably good value for ΔN_ν, even though it might not always be easy to decide whether a small peak must be considered as a resonance or not.

Another method is to use the asymptotic relation (Cremer *et al.* 1973)

$$\Delta N_\nu = 4m_\nu \operatorname{Re}\{1/Z_\nu\}\Delta f. \tag{6a}$$

Here Δf is the frequency range (in Hertz) of interest and Z_ν is the input impedance of the system, provided it is in the average not much influenced by boundary effects. $\operatorname{Re}\{1/Z_\nu\}$ may be taken from the literature 'for the corresponding infinite system', or it may be measured.

Figure 4 shows in its upper part on a logarithmic scale the SEA temperature that was measured on a three-plate rearrangement. Plate 2 was excited successively at five points, and 18 to 30 response locations per plate were used. All the measured input powers and energies were summed up. For the number of modes the asymptotic value for plates was taken.

In its lower part figure 4 gives the net energy flow $W_{\nu k}$ (see (8)) from one subsystem to another one using the methods described later. The power coming from outside was always $P_2^{(2)} = 1$ W, corresponding to 120 dB. Since W_{12}, W_{23}, W_{13} are net energy flows they may be positive or negative. To indicate their direction in the figure and still retain the logarithmic scale, positive and negative energy flows are plotted in

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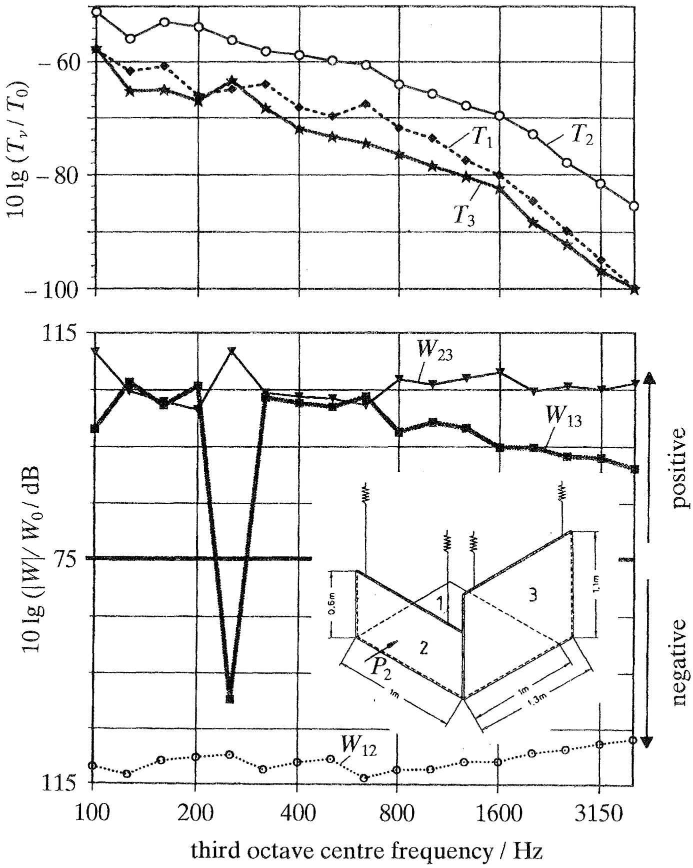

Figure 4. Temperature and net energy flow in a three plate system.

different directions. The range below 75 dB is missing (it is not known accurately anyway). The reference value always is $W_0 = 10^{-12}$ W.

It can clearly be seen that the direction of the SEA-measured energy flow agrees with the sign of the temperature difference although the plates did not have a high modal density. In the third octave centred at 200 Hz there were approximately three modes, in the 2000 Hz third octave about 32.

Figures 5 and 6 show the results of computer simulations. In this case the modal expansion of three coupled simply supported plates was used to calculate the mean square velocities of the plates. The parameter of the plates were chosen in such a way that the asymptotic number of modes was the same as in the example shown in figure 4 (0.058 to 0.064 modes Hz^{-1}). The velocities for three different types of excitation which were found this way were then introduced into (2) to obtain the net energy flow. In the example shown in figure 5, plate 1 was excited, but because there was a soft spring between 1 and 2 and a stiff spring between 1 and 3 the SEA temperature T_3 is higher than T_2 and consequently W_{23} is negative; i.e. there is a net energy transport from 3 to 2. With respect to the energy flow from 1 to 2 the temperature curves clearly indicate a flow from 1 to 2, which is quite plausible. The curve for W_{12}, however, gives partly positive and partly negative values. This is probably due to the fact that W_{12} is rather small and therefore its measurement is not very reliable. In computer simulated experiments, the energy flows can also be calculated directly to

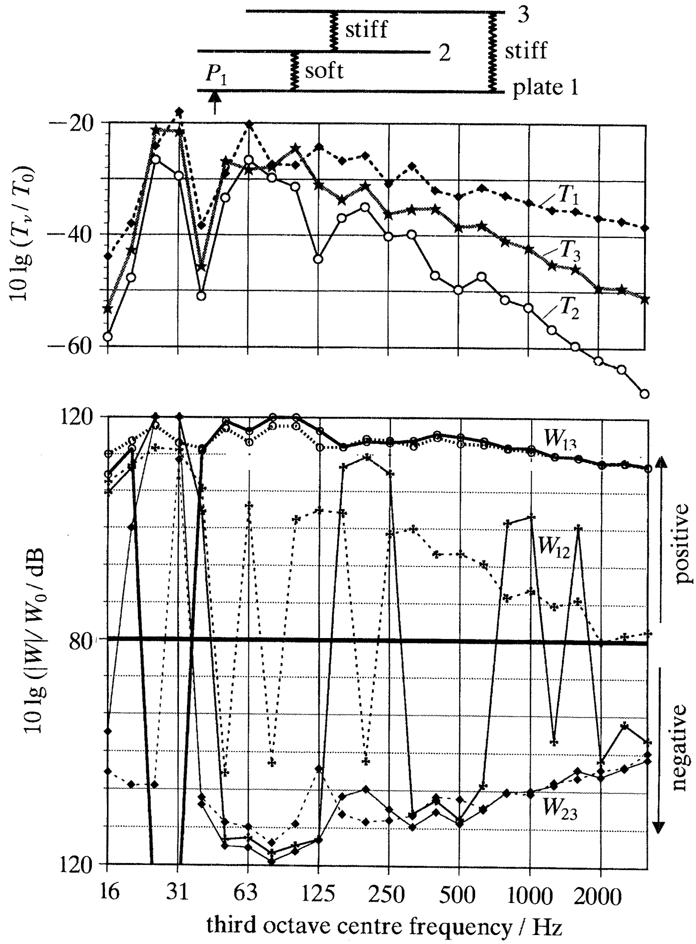

Figure 5. Temperature and net energy flow in a three plate system (computer simulation). Broken lines are the true values based on modal expansion. Continuous lines are the SEA calculations based on simulated experiment.

verify the SEA-measured values. They are included in figures 5 and 6, and it can be seen that the results are very good for the larger energy flows above 125 Hz.

In the example shown in figure 6 the centre plate 2 was excited and all coupling springs were very stiff. The SEA temperature curves indicate an energy flow from 2 to 3 and 2 to 1. This agrees with the curves for W_{23} and $W_{12} = -W_{21}$. Above 125 Hz T_1 is in general greater than T_3 and therefore W_{13} is mainly positive.

For frequencies below 125 Hz the curves for the net energy flow are rather erratic because in this frequency range the system is well coupled and there are only two or less modes in a third octave band.

In conclusion, this part of the paper shows that the measurement of the SEA temperature (based on the asymptotic number of modes) yields a reasonably good indication of the net energy flow direction. In addition, the SEA temperature distribution helps to find those subsystems that are strongly coupled because their SEA temperatures are more or less equal.

Measurements of the SEA temperature are especially useful when subsystems of completely different types are connected. Examples are the sound pressure in a small

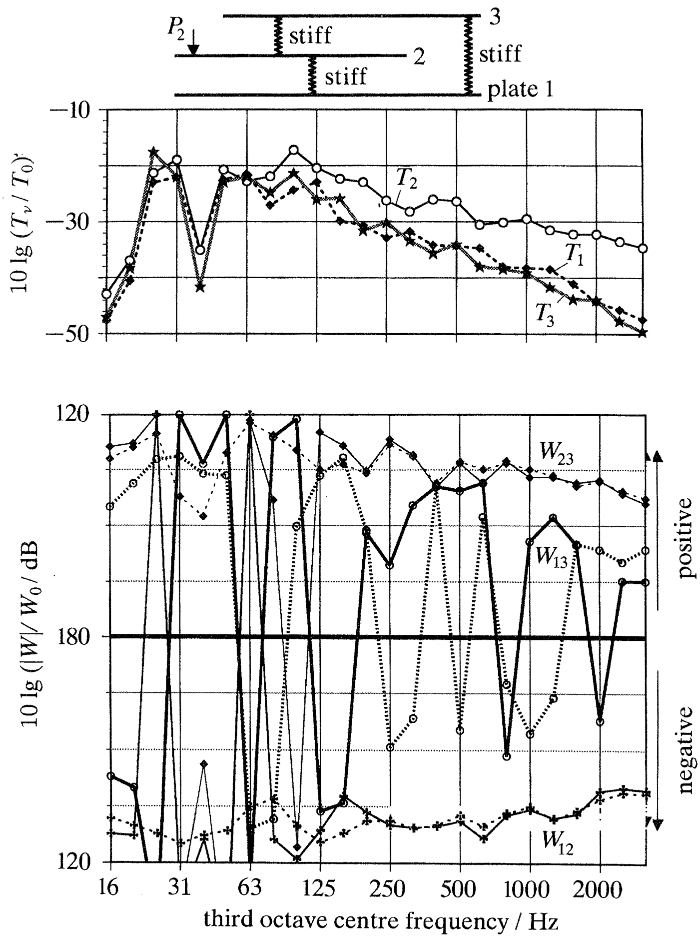

Figure 6. See figure 5.

space (e.g. the oil in a pipe) which is coupled to the vibrations of the surrounding walls, or the in-plane waves in a plate that are coupled to the bending waves of another (or the same) structure. In such cases the measured quantities such as root mean squared velocities, accelerations, or pressures cannot be compared, but the SEA temperatures can.

5. Measurements and computer simulations

(a) *Internal loss factors*

For the measurement of internal loss factors the SEA equations are added in such a way that all terms containing the coupling loss factors are eliminated. For a three subsystem arrangement (5d) is obtained this way. In the general case of n subsystems which are excited successively by n outside forces, the resulting set of equations is

$$\sum_{\nu=1}^{n} \eta_{\nu d} E_\nu^{(\mu)} = \frac{1}{\omega} P_{\mu,\,\mathrm{in}}^{(\mu)} \delta_{\nu\mu}. \tag{7}$$

Here the number of different excitation situations has to range from $\mu = 1$ to $\mu = n$.

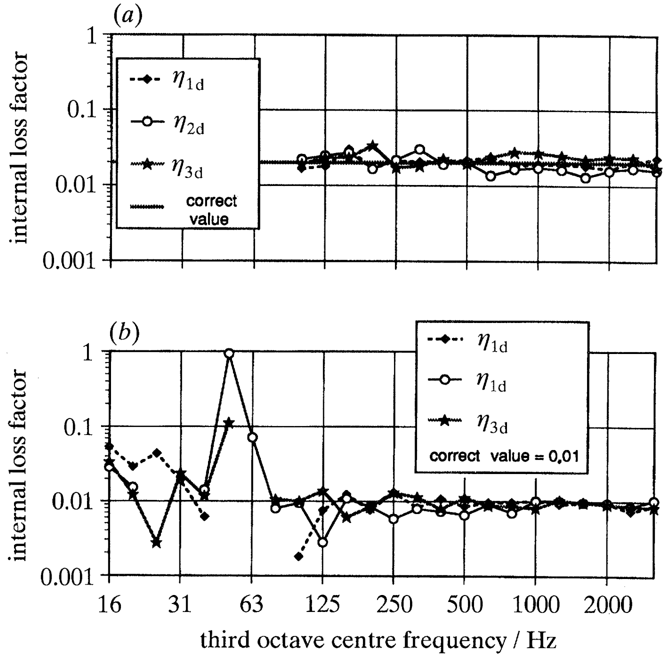

Figure 7. Internal loss factors calculated by using equation (7). (*a*) Real experiments using three plates. (*b*) Computer simulation. (In both cases there were less than two modes per frequency band below 125 Hz.)

This method was applied successfully by Ming *et al*. (1990). Other results are shown in figure 7. They are reported by Lewit & Lehmköster (1992).

In the 'real' experiment three wood fibre boards (see also figure 4), which had different sizes and thicknesses but the same loss factors, were rigidly connected at the edges and $\eta_{1d}, \eta_{2d}, \eta_{3d}$ was measured by using (7). The 'correct value', which is also given in figure 7, is based on the reverberation times at different frequencies of a free-free bar that consisted of the same material. The computer simulation in figure 7 is based on a three plate configuration. In this case the mean square velocities were calculated using a modal expansion. The results were then inserted into (7).

If one keeps in mind that the results are based on measured average levels that have an accuracy of 1–2 dB, the agreement is surprisingly good.

(*b*) *Net energy flow*

In (2) the energy flow between two subsystems is expressed in terms of the loss factor. But in the literature the basic SEA relations are also written as

$$\left.\begin{aligned} \omega\eta_{\nu d} E_\nu + \sum_{k\neq\nu} W_{\nu k} &= P_{\nu,\,\mathrm{in}} \\ \text{or}\qquad \eta_{\nu d} E_\nu + \sum_{k\neq\nu} \alpha_{\nu k}(T_\nu - T_k) &= \frac{1}{\omega} P_{\nu,\,\mathrm{in}}. \end{aligned}\right\} \tag{8}$$

Here $W_{\nu k}$ is the net energy flow from ν to k. The values T_ν and T_k are the SEA temperatures given by (6). A very important aspect for the application of (8) is the symmetry relation for the 'conductances' $\alpha_{\nu k}$; i.e.

$$\alpha_{\nu k} = \alpha_{k\nu}. \tag{9}$$

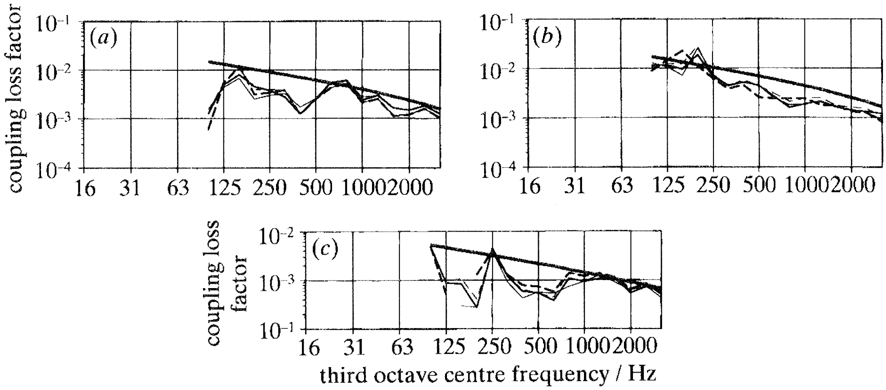

Figure 8. Comparison of measured and theoretical (thick) coupling loss factors. (*a*) η_{12}, (*b*) η_{13}, (*c*) η_{23}.

In the basic SEA theory this relation is proved for coupled oscillators. If (8) is applied to n different excitation situations, the result is

$$\eta_{\nu d} E_{\nu}^{(\mu)} + \sum_{k \neq \nu} \alpha_{\nu k}(T_{\nu}^{(\mu)} - T_{k}^{(\mu)}) = \frac{1}{\omega} P_{\mu,\,\mathrm{in}}^{(\mu)} \delta_{\nu\mu}. \tag{10}$$

Here $P_{\mu,\,\mathrm{in}}^{(\mu)}$ is again the power that is transmitted from outside into the μth subsystem (and only into this one) in the μth experiment.

Equation (10) is a system of $n \times n$ equations. By adding those relations that have the same value of μ, we again find (7). Because we used this relation already for finding the internal loss factors, we can exclude n equations from (8). For the sake of simplicity we choose to exclude those relations that have the incoming powers on their right-hand side. This way we are left with $n(n-1)$ relations that do not contain the incoming powers (which are sometimes hard to measure); i.e.

$$\sum_{k \neq \nu} \alpha_{\nu k}(T_{\nu}^{(\mu)} - T_{k}^{(\mu)}) = -\eta_{\nu d} E_{\nu}^{(\mu)}. \tag{11}$$

Here ν and μ range from 1 to n but $\nu = \mu$ is excluded. Because of the symmetry relation $\alpha_{\nu k} = \alpha_{k\nu}$, equation (11) is an overdetermined system of $n(n-1)$ equations for $\frac{1}{2}n(n-1)$ unknowns. Thus we would be allowed to exclude half of the equations in (11). But as there is no simple rule for deciding which should be retained, it is better to write (11) as

$$\sum_{k \neq \nu} \alpha_{\nu k} D_{\nu k}^{(\mu)} = -\eta_{\nu d} \Delta N_{\nu} \tag{12}$$

and to solve this overdetermined system by a least square technique. In (11) the relative temperature difference

$$D_{\nu k}^{(\mu)} = (T_{\nu}^{(\mu)} - T_{k}^{(\mu)})/T_{\nu}^{(\mu)} \tag{13}$$

was introduced, where ΔN_{ν} is again the number of modes in the νth subsystem. The relative temperature difference constitutes a normalization which helps to avoid $P_{\mu,\,\mathrm{in}}$ having a strong influence on the final results. Figure 8 shows results that were obtained by this and other methods from the setup introduced in figure 4. The quantity plotted in figure 8 is $\eta_{\nu k} = \alpha_{\nu k}/\Delta N_{\nu}$.

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Apart from experiments with the real three plate arrangement, several computer simulations were made using the model that is briefly described in connection with figures 5 and 6. By using the modal expansion the 'true' net energy flow could be calculated and compared with the energy flow that is found from (12). The result is shown in the upper part of figure 10. It can be seen that there is good agreement between the 'true value' and the SEA calculation based on simulated experiments.

(c) *Coupling loss factors*

If the SEA equations are applied to several excitation situations (e.g. (2)) it is useful to distinguish between the following two cases:

1. No assumption is made with respect to a relation between the coupling loss factors $\eta_{\nu k}$ and $\eta_{k\nu}$. In this case one has to solve the full set of linear equations

$$\eta_{\nu d} E_{\nu}^{(\mu)} + \sum_{k \neq \nu} (\eta_{\nu k} E_{\nu}^{(\mu)} - \eta_{k\nu} E_{k}^{(\mu)}) = \frac{1}{\omega} P_{\mu,\,\mathrm{in}}^{(\mu)} \delta_{\nu\mu} \tag{14}$$

with $1 \leqslant \nu \leqslant n$, $1 \leqslant \mu \leqslant n$. It can be seen that there are n^2-equations and the same number of unknowns. Thus the system can be solved either directly or after making the Lalor-rearrangement that was used in (5*a*–*d*).

2. If, however, the reciprocity relation

$$\eta_{\nu k} \Delta N_{\nu} = \eta_{k\nu} \Delta N_{k}, \tag{15}$$

which is rather fundamental in SEA, is applied, the number of unknowns is reduced to $n + \frac{1}{2}n(n-1)$, because, apart from the n values for $\eta_{\nu d}$, we have to find only $\frac{1}{2}n(n-1)$ coupling loss factors $\eta_{\nu k}$.

There are several ways to treat this overdetermined system.

(i) Because the absolute measurement of $P_{\mu,\,\mathrm{in}}$ is generally of limited accuracy, all equations containing this quantity are excluded (see Lewit & Petit 1991). This way an overdetermined, homogeneous system of equations is obtained. As coefficients it contains the energy ratios E_k/E_ν. With a least square technique

$$\eta_{\nu d}/\eta_{\mathrm{ref}} \quad \text{and} \quad \eta_{\nu k}/\eta_{\mathrm{ref}}$$

can be found. The quantity η_{ref}, for which one may take η_{1d} or any other loss factor, is the only loss factor that cannot be calculated by this method which is based on a homogeneous set of equations. Usually this is not a serious drawback because relative loss factors contain most of the desired information. If this should not be the case, at least one non-zero loss factor has to be found by an independent method.

(ii) The assumption (15) is equivalent to setting

$$\eta_{\nu k} = \alpha_{\nu k}/\Delta N_{\nu}. \tag{16}$$

This way – except for a factor – equation (12) is obtained and the methods described there can be applied.

(iii) In principle one could exclude $\frac{1}{2}n(n-1)$ equations from (14). But it is hard to decide what can be excluded, without getting an ill-conditioned matrix or other accuracy problems; therefore this method is not discussed further here.

Figure 8 shows results that were obtained by using the different methods. The data are so close together that it did not seem necessary to indicate which curve belongs to which method. In addition, figure 8 also gives a theoretical solid line (W. Wöhle, personal communication). It is based on the theoretical transmission coefficient of two infinite plates that are connected at right angles.

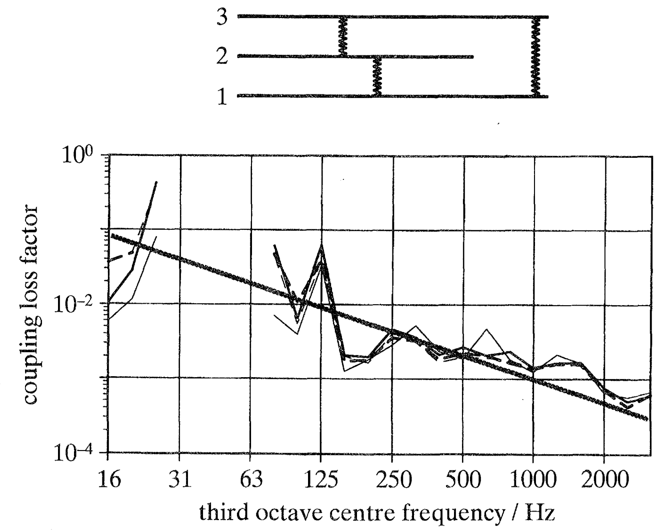

Figure 9. Comparison of coupling loss factors derived from computer simulations and theory (thick line). Internal loss factor 0.01.

In figure 9 the results of computer simulations are plotted. In this case three plates already introduced in figure 6 were coupled by rigid connectors; therefore a theoretical coupling loss factor can be derived from Cremer's (1953) formula for sound bridges between two plates. The result is

$$\eta_{\nu k} = \frac{1}{2|A|^2 \omega m_\nu} \operatorname{Re}\left\{\frac{1}{Z_k}\right\}, \quad \text{with} \quad A = \frac{1}{Z_\nu} + \frac{1}{Z_k}.$$

Here Z_ν, Z_k are the impedances of subsystems ν and k if they were infinite.

When the experimental data and the theoretical curve in figures 8 and 9 are compared, one is confronted with the question whether the discrepancies are due to shortcomings of the inverse SEA method or due to the inapplicability of the theoretical data to the experimental situation.

It is our opinion that – even above 160 Hz when there are at least three modes per third octave and the systems are weakly coupled – the theoretical data do not adequately describe the experimental situation and therefore the discrepancies do not indicate shortcomings of the method. The reasons for this opinion are as follows.

(*a*) The general trend of the theoretical and experimental curves is the same.

(*b*) Measurements (which are not repeated here) showed that within 1–2 dB the measured coupling loss factors obeyed the reciprocity relation $\eta_{12} \cdot \eta_{23} \cdot \eta_{31} = \eta_{21} \cdot \eta_{32} \cdot \eta_{13}$ (obviously this test was only made when (15) was not used).

(*c*) The four different methods (see figure 9) gave practically the same results.

(*d*) In the computer simulation the modal expansion allows to calculate the 'true' net energy flow between two finite systems. It can also be used to simulate the inverse SEA method which is based on squared quantities. In figure 10*a* comparison is made for the arrangement from figure 6 but with very low internal loss factors (0.0001). It shows that the net energy flow W_{12} is determined rather exactly although the measured coupling loss factors do not agree with the theoretical values. Thus agreement with theoretical values that are based on infinite substructure transmission coefficients cannot be required.

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

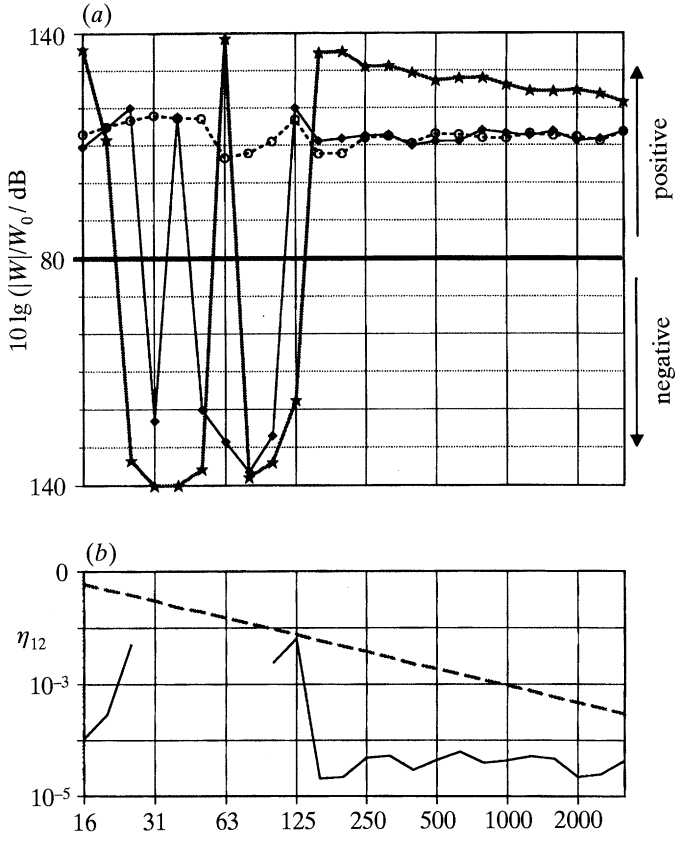

Figure 10. (*a*) Net energy flow, W_{12}, of a three plate arrangement (computer simulation). ○, True value based on modal expansion; ◆, SEA calculation based on simulated experiment; ★, SEA calculation based on 'theoretical' coupling loss factor. (*b*) Coupling loss factor, η_{12} of a three plate arrangement. ——, Result of simulated SEA calculation; – – –, 'theoretical' (infinite) value. Internal loss factor 0.0001.

(*e*) For an investigation of the applicability of 'theoretical' values it is very revealing to study the influence of internal damping on the coupling loss factors. There are two limiting cases: (i) when two simple degree of freedom resonators are coupled, the coupling loss factors depend strongly on internal damping (see Lyon 1975 or Cremer *et al.* 1973); (ii) when two infinite subsystems (infinite number of modes) are coupled, internal damping has only a second order influence on the coupling loss factor. Practical situations are somewhere in between and indeed it could be shown by computer simulations, that the coupling loss factors depend on the internal damping of the receiving substructure whenever $\eta_{\nu k} \gg \eta_{\nu d}$; i.e. when there is strong coupling. The lower part of figure 10 gives an example of this type; it shows an 'experimental' coupling loss factor which is well below the one plotted in figure 9 because the internal loss factors are much smaller.

The upper part of figure 10 shows that the 'theoretical' coupling loss factor would even lead to a net energy flow W_{12} that is larger than the incoming power of 1 W → 120 dB.

The examples shown in most of the graphs indicate that SEA allows us to quantify

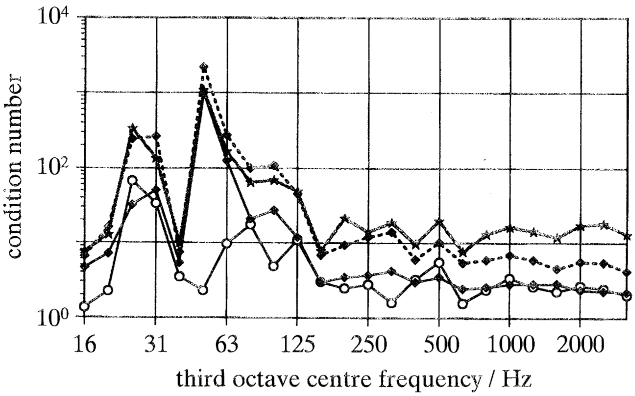

Figure 11. Condition number for different methods of measuring coupling loss factors. —◆—, Equation (14) without relations containing $P_{\mu}^{(\mu)}(n_E = 6, n_u = 6)$; —○—, equation (11) with $\eta_{\nu d}$ known ($n_E = 6, n_u = 3$); ★, equation (14) combined with equation (15) ($n_E = 6, n_u = 6$); —◆—, equation (5*a*–*d*) ($n_E = 9, n_u = 9$). n_E = number of equations, n_u = number of unknowns.

the transmission paths in a complex system. But obviously there are also drawbacks. Apart from the rather large experimental effort, a fundamental one is that small net energy flows may be buried under larger ones. Figure 5 (lower part) contains an example of this type. Here the curve for W_{12} is rather erratic (and in disagreement with the true energy flow calculation) although the temperatures clearly show that energy must flow from plate 1 to plate 2. This result – and others not reported here – shows difficulties that arise when there is a small energy flow, derived from a small coupling loss factor, in the presence of a large energy flow from somewhere else into the same subsystem. But as one is usually interested mainly in the important transmission paths this is not too great a disadvantage.

(*d*) *Condition number*

Whenever one is dealing with an inverse problem, the question of ill-posedness comes up. For the case of linear equations as they appear in SEA, a problem is ill-posed when the condition number is high (see Stewart 1973). Thus it seems useful to accompany calculations of the type reported here with the determination of the condition number. Figure 11 shows several examples. It can be seen that at low frequencies, when there are only a few modes, the condition number became high; in the remaining frequency range it was between 1 and 10.

It does not seem possible to give a fixed upper limit for the condition number. But at least one can say that an error of $p(\%)$ in the input data leads at worst to an error of $C_n\,p(\%)$ in the final result (C_n is the condition number). Thus for $C_n = 10$ a 1 dB (= 25 %) error in the input data leads, in the worst case, to a result that deviates by 250 % (= 4 dB) from the correct value. Such errors may appear high but, because they are worst cases, it seems reasonable to assume that with some care the average accuracy of the final results can be brought to 2–3 dB.

6. Conclusions

The ‘inverse’ use of SEA constitutes a useful method for gaining information about the transmission paths in complex structures.

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

If only the direction of sound and vibration transmission is of interest, it is sufficient to determine the SEA temperatures according to (6). The net energy flow always is from the higher temperature to the lower one.

If two adjacent subsystems independent of source position have the same temperature, they are well coupled. Temperatures can be compared even if they are based on the mean square velocities of different wave types or on pressures in different fluids or gases.

When SEA is used to calculate the net energy flow, or the internal loss factor, or the coupling loss factors in a given complex system, then several experiments are required. They involve the excitation with different source configurations and the measurement of many mean square velocities (or pressures). The data obtained this way are used to establish a set of linear equations for the unknown quantities. There are several ways for solving this set of equations, especially when they form an overdetermined system. This is the case when the symmetry relations (9) or (15) are used or when certain quantities are known *a priori* from other independent measurements.

Practical applications and computer simulations on arrangements consisting of three plates generally gave good results when there were more than three modes in a frequency band of interest and when the coupling was weak. Poor results for the coupling quantities were obtained when (i) the corresponding subsystems were well coupled (i.e. small temperature difference), (ii) the total energy flow into a certain subsystem originates to a small part from one neighbouring system and to a large part from another one. In the first case the coupling loss factor is not the appropriate way to describe the situation (the temperature distribution would be better), the second case is not of great practical relevance.

Comparison of measured coupling loss factors with theoretical values that are based on calculation of the transmissibility of infinite substructures, should be made only when the subsystems are not well coupled, i.e. when they have sufficient damping. The conditions underlying such theoretical data are much more stringent than the conditions for the applicability of SEA.

The condition number of the matrix that has to be inverted always should be calculated, because it gives a good indication of the error sensitivity of the results.

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PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES